Analog and Digital Representation

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Abstract: The distinction between analog and digital representation is central to recent debates about numerical cognition. However, there are certain aspects of the analog-digital distinction that have gone largely unnoticed, and yet their clarification may affect the contours of debate. My aim is to clarify those aspects of the distinction. In doing so, I argue for three claims. The first is that while it is commonly held that analog representations are *continuous* and that digital representations are *discrete*, properly understood the distinction concerns the *format* and not the *medium* of representation. The second is that sometimes the distinction turns on facts about the *user* of a system of representations. The third is that these two claims are implicit in Haugeland's (1998) account of the analog-digital distinction.

Introduction

The distinction between analog and digital representation is central to recent debates about numerical cognition. In particular, psychologists have documented a range of approximate numerical capacities that are possessed by human beings of various ages,[1] and in order to explain those capacities, many have posited a system of *analog magnitudes*, arguing that a *digital* system of representation could not account for the experimental data.[2] However, there are certain aspects of the analog-digital distinction that have gone largely unnoticed, and yet their clarification may affect the contours of debate. My aim, therefore, is to clarify those aspects of the distinction. In doing so, I argue for three claims.

The first is that while it is commonly held that analog representations are *continuous* and that digital representations are *discrete*, properly understood the distinction concerns the *format* and not the *medium* of representation. The second is that sometimes the distinction turns on facts about the *user* of a system of representations.

[1] These capacities are present early in infancy, and remain present in older children and adults, long after the acquisition of precise numerical capacities. Moreover, approximate numerical capacities similar to those observed in humans have been documented in a range of other species.
[2] See for example, Wynn (1992), Dehaene (1997), Laurence and Margolis (2005), and Gallistel, Gelman and Cordes (2006).

The third is that these two claims are implicit in Haugeland's (1998) account of the analog-digital distinction.

1. Medium and Format

Consider two systems for representing the natural numbers. System A employs a large unmarked beaker and a supply of water, which is to be poured into the beaker in equivalently-sized increments. If the beaker contains *n* increments, it represents the number *n*.

System B employs a supply of marbles. The set of marbles is partitioned according to color, and there are ten colors. There is a one-to-one function associating marbles and the numbers zero through nine, such that the color of a marble *m* determines the number with which that marble is associated, $f(m)$. Numbers are represented by arrays of marbles. The number represented by an array of length *n* is $R(arr_n) = \sum R(pv_i)$ for $i = 1 \ldots n$, where $R(pv_i) = f(m_i) \times 10^{i-1}$, where m_i is the marble in the *i*th place from the right. This is the familiar decimal system, using colored marbles instead of Arabic numerals.

Many will want to say that system A is analog and that system B is digital. That intuition is likely due to the view that analog representation is continuous while digital representation is discrete. Since water is (or at least appears) continuous, system A is analog. Since marbles are discrete, system B is digital. But consider two more systems.

System C is constructed using a supply of water and a supply of beakers. Water is poured into the beakers in equivalently-sized increments, such that each beaker contains between zero and nine increments of water. There is a one-to-one function associating beakers with the numbers zero through nine, such that the amount of water in a beaker *b* determines the number with which that beaker is associated, $f(b)$. Numbers are represented by arrays of beakers. The number represented by an array of length n is $R(arr_n) = \sum R(pv_i)$, for $i = 1\ldots n$, where $R(pv_i) = f(b_i) \times 10^{i-1}$, where b_i is the beaker in the ith place from the right.

System D employs a large unmarked beaker and a supply of marbles. The marbles are to be poured into the beaker in equivalently-sized increments. An increment contains a large approximate number of marbles (for example, about one hundred). If the beaker contains n increments, it represents the number n.

Call whatever stuff from which representations are constructed the *medium of representation*. Call whatever structure is imposed on that medium the *format of representation*. Systems A and C employ the same medium of representation: water. Systems B and D also share a medium of representation: marbles. But systems A and D have the same format of representation, as do systems B and C. And while system A may be analog and system B digital, system C is digital and system D is analog. Hence, the analog-digital distinction concerns the format of representation, and not the medium of representation.

2. Appearing Continuous and Appearing Discrete

Representational systems are (typically) employed by someone or something. Whatever it is that employs a system of representations, in a given case, I will call the *user* of that system. The user is whatever part of the mechanism "reads" and "writes" the representations. In the above examples, whoever or whatever pours water or marbles into the beakers, or places marbles or beakers of water into arrays, is the user of those systems.

Consider again system D, and suppose the user is a human being. Because a large number of marbles are employed in each increment, the user will likely be unaware exactly how many marbles are employed in any given representation. Because of this, the user is likely to be unable to readily discern whether a representation is of some number *n*, or whether it is of some other relatively nearby number. This will especially be so if the number represented is particularly large. In other words, given two token representations, *t* and *t′*, the user will be unable to readily determine whether or not *t* and *t′* are of the same representational type.

Just the opposite is the case with system C. While the user cannot readily distinguish elements of the medium of representation (individual water molecules) the user can readily distinguish representations of some number *n* from representations of other nearby numbers. That is, given two token representations, *t* and *t′*, the user will be able to readily determine whether or not *t* and *t′* are of the same representational type.

One way of expressing this difference is in terms of how the representations appear to the user of the system. System C's representations appear continuous to the user. System D's representations appear discrete to the user. This then is the sense in which analog representations are continuous and digital representations are discrete. The former appear continuous – the user cannot readily distinguish them from each other. The latter appear discrete – the user can readily distinguish them from each other.

On this account, whether or not a representational system is analog or digital may turn on facts about the user of the system. For example, if the perceptive powers of a particular human being were sufficiently enhanced, then the representations of system D would be readily distinguishable to that user. That is, the representations would appear discrete to that user. It would therefore be a digital, rather than an analog system.

3. Devices

John Haugeland (1983) has offered what is perhaps the most detailed philosophical account of the analog-digital distinction.[3] I want to show that it implicitly suggests the account just given. Haugeland's is a general account of the distinction between analog and digital "devices," where he is "noncommittal" about what qualifies as a device. It is thus intended as an account of the distinction as applied to anything. As the present concern is with systems of representation, I will extract an account of digital and analog representation from his account of digital and analog devices.

Haugeland claims that digital devices are defined by the following four features:

(i) a set of types,

(ii) a set of feasible procedures for writing and reading tokens of those types, and

(iii) a specification of suitable operating conditions, such that

(iv) under those conditions, the procedures for the write-read cycle are positive and reliable. (78)

Here a procedure is positive just in case it "can succeed absolutely and without qualification," and a procedure is reliable just in case "under suitable conditions, [it] can be counted on to succeed virtually every time" (77).

[3] See also von Neumann (1958), Goodman (1968), and Lewis (1971).

Haugeland thinks of analog devices as employing write-read cycles, as do digital devices, but he thinks of these procedures as “approximation” procedures that are defined by margins of error, such that:

(v) the smaller the margin of error, the harder it is to stay within it,

(vi) available procedures can (reliably) stay within a pretty small margin,

(vii) there is no limit to how small a margin better (future, more expensive) procedures may be able to stay within, but

(viii) the margin can never be zero – perfect procedures are impossible (83).

Since Haugeland allows that analog devices employ representational types and write-read cycles, it should follow that analog devices also have the features (i)-(iv), but with a modification on (iv). In particular, in the case of analog devices, it should read as follows:

(iv_a) under those conditions, the procedures for the write-read cycle are approximate and reliable.

And here the notion of “approximate” is given by the notion of a margin of error, and (v)-(viii) explain the relationship between a margin of error and the reliability of the write-read cycle. In general, the smaller the margin of error, the less reliable the process.

Thus, digital devices have the features (i)-(iv), and analog devices have the features (i)-(iii) and (iv_a). Again, the present concern is with systems of representation. Presumably an analog device is one that employs analog representations, and a digital device is one that employs digital representations. Thus digital representations are those used by devices that have the features (i)-(iv), and analog representations are those that are used by devices that have the features (i)-(iii) and (iv_a).

If this is right, then an analog device may be converted to a digital device, by increasing the margin of error such that the write-read process becomes a positive procedure, rather than an approximate one. Haugeland writes that, “all ordinary (and extraordinary) procedures for parking the car right in the center of the garage, cutting six-foot boards, measuring out three tablespoons of blue sand, and copying photographs, are approximation procedures” (84). But the reason he casts these as approximation procedures is because he assumes a margin of error of zero. So for example, it would take if not infinitely small measurements, certainly measurements well beyond what is presently possible to discern whether the car is parked exactly in the middle of the garage. If a larger margin of error is adopted though (for example, within three feet of the center of the garage) then there are positive procedures for parking the car in the center of the garage.

Note also that the idea that both analog and digital devices involve write-read cycles is essentially the idea that they involve a user. For the user just is whatever (or whoever) writes and reads the tokens. Moreover, whether a given margin of error is large enough to allow for positive procedures may turn on facts about the user of the representations.

For example, suppose we have a device in which the representational types are these: the car is parked such that its midline is within three feet of the center of the garage, the car is parked such that its midline is more than three feet to the left of the center of the garage, and the car is parked such that its midline is more than three feet to the right of the center of the garage. Suppose also that the user is a human being able to reliably park the car such that its midline is within two feet of any desired spot in the garage (barring of course, areas too close to the walls), and who is able to reliably determine that the car is so parked. In this case, the margin of error allowed for is large enough that the procedures involved are positive and reliable. Thus the system is digital.

In contrast, suppose that we have a device in which the representational types are these: the car is parked such that its midline is within one quarter inch of the center of the garage, the car is parked such that its midline is between one and three quarters of an inch to the left or right of the center of the garage, the car is parked such that its midline is between three quarters of an inch and one and one quarter of an inch to the left or right of the center of the garage, and so on so that every half an inch constitutes a new representational type. Suppose also that the user is the same human being as above. Now the procedures are neither positive nor reliable, and thus, the system is analog.

Finally, notice that what makes the first system digital – what makes it the case that its processes are positive and reliable – is that the user is able to readily determine whether two token representations are of the same type or not. What makes the second system analog – what makes its processes neither positive nor reliable – is that the user cannot readily determine whether two tokens are of the same type or not. That is, in the former case the representations appear discrete to the user, while in the latter case they appear continuous to the user.

4. Conclusion

In short, analog representations appear continuous to their user, and digital representations appear discrete to their user. This concerns the format, and not the medium, of representation. Moreover, whether a given representational system is analog or digital will sometimes turn on facts about the user of that system. Finally, this view is implicit in Haugeland's account of analog and digital devices.

These conclusions should have impact on our understanding of numerical cognition. For the presence of approximate numerical capacities in human infants demands postulation of a system of numerical representation, and that has suggested to some a foundation for later, more precise numerical knowledge.[4] But the approximate nature of the capacities demands an analog system of representation, which has been taken to imply a continuous system of representation, which in turn has been taken to imply difficulties in explaining how the system might develop, so as to be capable of explaining the acquisition of later, more precise capacities.[5] But an analog system implies only the appearance of continuity, not actual continuity, and that difference may in the end allow for an explanation of how an analog system can account for precise numerical capacities.

[4] See for example, Spelke (2003), Hauser and Spelke (2004).
[5] See for example, Laurence and Margolis (2005).

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